MODELING NONISOTHERMAL STEADY FLOW OF A REAL GAS IN A PIPE ON AN ANALOG COMPUTER

V. B. Krivonosov and B. L. Krivoshein

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The authors present a procedure for modeling nonisothermal steady flow of a real gas in a pipe on an MNB-1 analog computer. Specific results are presented.

Calculations of nonisothermal flows of a real gas in pipes are of practical interest in determining zones of hydration and hydrocarbon and water-vapor condensation, in defining more accurately the mean gas temperature, etc. As is well known [1], in general, the steady nonisothermal flow of a real gas in pipes is described by a system of equations of conservation of mass, linear momentum, energy, and state. If we introduce assumptions that are justified for subsonic velocities in long pipes [1] (variation of the velocity head and the difference between the bench marks at the ends of the pipe can be ignored) and take the law of heat transfer of the gas pipe with the atmosphere through the ground in Newton's form, we can reduce the above-mentioned equations to a system of two nonlinear ordinary differential equations [2]:

$$\frac{dP}{dx} = -a\frac{ZT}{P},$$
$$\frac{dT}{dx} = -b\frac{T^3 Z}{P^2}\left(\frac{\partial Z}{\partial T}\right)_p + h(T_0 - T). \qquad (1)$$

System (1) is nonlinear; therefore, its solution in finite analytic form is obviously impossible and we must use either approximate methods (for example, [2]) or analog techniques. Finite results can be obtained on digital computers with high accuracy, but preparation of the problem often requires great expenditures of time and effort. Modeling methods can be successfully used to solve this problem. With all of their visualizability and ease in setting up a problem, analog devices have fairly high engineering accuracy. In the given case, the accuracy of the solution was established by comparing the results with numerical calculations.

System (1) is reduced to dimensionless form for simplicity of calculation and for convenience in modeling the problem. The compressibility of the gas is taken according to Berthelot's equation [3] $\bar{Z} = 1 + (9/128)(1 + 6/\bar{T}^2)\bar{P}/\bar{T}$ and the dimensionless variables are $\bar{x} = x/L$, $\bar{T} = 10T/T_{cr}$, $\bar{T}_0 = 10T_0/T_{cr}$, and $\bar{P} = 10P/P_{cr}$.

$$\frac{d\bar{P}}{d\bar{x}} = -\gamma\frac{\bar{T}}{\bar{P}} + \frac{54}{1.28}\gamma\frac{1}{\bar{T}^2} - \frac{9}{128}\gamma,$$

$$\frac{d\bar{T}}{d\bar{x}} = 1800\,\theta\frac{1}{\bar{T}^2}\frac{d\bar{P}}{d\bar{x}} - \theta\frac{d\bar{P}}{d\bar{x}} - h_1\bar{T} + h_1\bar{T}_0, \qquad (3)$$

where

$$\gamma = \frac{10LaT_{cr}}{P_{cr}^2};\quad \theta = \frac{9}{128}\frac{R}{c_p};\quad h_1 = \frac{K\pi LD}{c_p G}.$$

The circuit according to which system (2) is set up on the analog computer is shown in Fig. 1. From it we set up the machine (modeling) equation:

$$p_m\varphi = -10k_3k_4\frac{\psi}{\varphi} + k_2k_4m\frac{1}{\psi^2} - k_1k_4U_0,$$
$$p_m\psi = \frac{1}{100}mk_5k_8(p_m\varphi)\frac{1}{\psi^2} - k_6k_8p_m\varphi - k_8k_9\psi + k_7k_8U_0. \qquad (2)$$

Here, φ and ψ are the voltages that model the pressure and temperature, respectively; t is the machine time, which simulates the coordinate; $p_m = d/dt$ is the differentiation operator; $k_1,\ldots,k_9$ are the transmission coefficients; U_0 is 100 V dc; and m is the scale of the function $1/\psi^2$.

We substitute into the machine equation the scale factors, which link the variables of the modeled and modeling systems of equations $m_1 = \bar{P}/\varphi$; $m_2 = \bar{T}/\psi$; $m_3 = \bar{x}/t$:

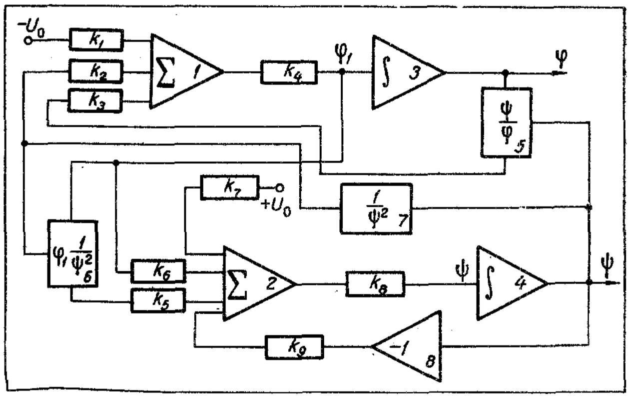

Fig. 1. Diagram for solving system of differential equations describing steady motion of a real gas through a pipe: 1, 2) adders; 3, 4) integrating units; 5, 6) multiplication-division units; 7) diode function generator; 8) inverter.

$$\frac{d\overline{P}}{d\overline{x}} = -10\,k_3k_4\,\frac{m_1^2}{m_2m_3}\,\frac{\overline{T}}{\overline{P}} + k_2k_4\,\frac{mm_1m_2^2}{m_3}\,\frac{1}{\overline{T}^2} - k_1k_4U_0\,\frac{m_1}{m_3},$$

$$\frac{d\overline{T}}{d\overline{x}} = \frac{1}{100}\,k_5k_8\,\frac{mm_2^3}{m_1}\,\frac{1}{\overline{T}^2}\,\frac{d\overline{P}}{d\overline{x}} - k_6k_8\,\frac{m_2}{m_1}\,\frac{d\overline{P}}{d\overline{x}} - k_8k_9\,\frac{\overline{T}}{m_3} + k_7k_8U_0\,\frac{m_2}{m_3}. \quad (4)$$

The identity of systems (2) and (4) is determined by the following equations:

$$10\,k_3k_4\,\frac{m_1^2}{m_2m_3} = \gamma; \quad k_2k_4\,\frac{mm_1m_2^2}{m_3} = \frac{54}{1{,}28}\,\gamma;$$

$$k_1k_4U_0\,\frac{m_1}{m_3} = \frac{9}{128}\,\gamma;$$

$$\frac{1}{100}\,k_5k_8\,\frac{mm_2^3}{m_1} = 1800\,\theta; \quad k_6k_8\,\frac{m_2}{m_1} = \theta;$$

$$\frac{k_8k_9}{m_3} = h_1; \quad k_7k_8U_0\,\frac{m_2}{m_3} = h_1\overline{T}_0.$$

This system determines all scale factors and transmission coefficients. The factors and coefficients that must be assigned are chosen on the basis of the specific conditions of the problem and the constraints imposed by the analog computer.

Figure 2 shows the results of calculations on an MNB-1 analog computer and on a BESM-2M digital computer. The calculations were made for a gas main with the following parameters: G = 100 kg/sec, $P_{cr} = 44.9 \cdot 10^5$ N/m², $T_{cr} = 190.6°$ K, $T_0 = 275°$ K, $\lambda = 0.012$, K = 2.3 W/m²·deg, D = 0.7 m, L = 10^5 m, $c_p = 2100$ J/kg·deg, $P_i = 53.9 \cdot 10^5$ N/m², and $T_i = 320°$ K.

The calculations on the BESM-2M were made by V. P. Radchenko by the Runge-Kutta method with a constant step. The results obtained on the MNB-1 differ from those obtained on the BESM-2M by not more than 1.0%.

Thus, the accuracy of calculation in a problem of nonisothermal gas flow on an analog computer is sufficient for engineering applications. The proposed method can, along with numerical and approximate methods, be used to calculate the nonisothermal motion of real gas in pipes.

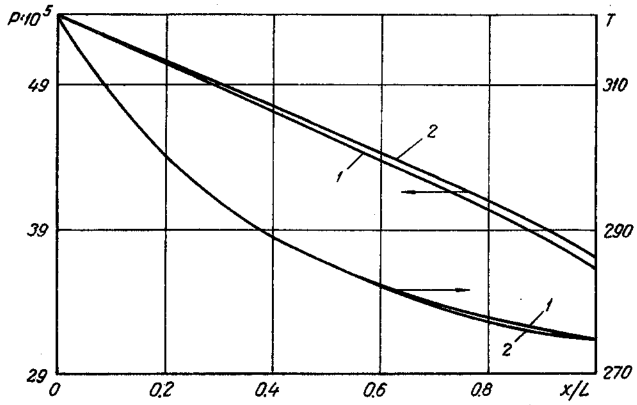

Fig. 2. Distribution of gas pressure (N/m²) and temperature (°K) along pipe: 1) calculated on MNB-1 analog computer; 2) on BESM-2M digital computer.

NOTATION

P is the pressure; T is the gas temperature; T_0 is the ground temperature at the depth of tube axis; Z is the gas compressibility coefficient; a, b, and h are the constant coefficients; G is the mass flow rate; D and f are the diameter and cross-sectional area of tube; L is the length of gaspipe; λ is the hydraulic resistance coefficient; K is the heat-transfer coefficient; c_p is the heat capacity of gas at constant pressure; T_{cr} is the critical temperature; P_{cr} is the critical pressure; R is the gas constant.

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Gubkin Institute of the Petrochemical and Gas Industry, Moscow